CS 228T QUIZ 2

Hints. Question 1 requires some very minor calculations, and a very small amount may help (but not required) in 4a, but the other questions don't involve any at all. If you find yourself doing math, stop and revisit the reading.

1. Suppose we have a weighted six-sided die and p_j, $j = 1, 2, \ldots 6$, is the probability of the jth side appearing when the die is tossed. For instance, p_6 is the probability of rolling a six. In addition, suppose we know that the average result from rolling the die is 4.7.

(a) State the optimization problem you would need to solve to find the maximum entropy die weights p_j consistent with this evidence.
(b) Write down the system of equations you would need to solve to actually obtain the solution to this problem. The equations can have Lagrange multipliers in them but should not contain unevaluated derivatives. (You do *not* have to actually solve for p_j.)

2. In the discussion of the clique tree algorithm for exact inference, we worked with a generic set of factors Φ over a set of variables X, where each factor $\phi_i \in \Phi$ has some scope $X_i \subset X$.

(a) In a Bayesian network G without evidence, what form do these factors take?
(b) Once we have a calibrated clique tree for G, what form do the clique beliefs take?
(c) Give the standard Bayesian network form of the distribution of the Markov chain $A \to B \to C$ as well as the clique tree-based reparameterization.

3. Suppose we have a model with random variables z and w, and w is observed and z is unobserved (latent or hidden). Consider the posterior of the hidden variable, given by

$$p(z \mid w) = \frac{p(z, w)}{p(w)}.$$

(a) Write down $D(q(z) \mid\mid p(z \mid w))$ in terms of the free energy functional.
(b) What quantity from the model does the free energy functional from part (a) provide a lower bound for?

4. State whether the following are true or false. (You don't need to give proofs or any other justification.)

(a) $D(q||p)$ is a convex function in q.
(b) The marginal polytope is a convex set.
(c) The local consistency polytope is a convex set.
(d) The local consistency polytope has more facets than the marginal polytope.
(e) The exact inference problem (CTree-Optimize-KL) is a convex optimization problem.
(f) The loopy BP problem (CGraph-Optimize) is a convex optimization problem.

Date: September 29, 2020.